

THE GENERALIZED SECOND LAW AND THE EMERGENT UNIVERSE

D. PAVÓN*

*Departamento de Física, Universidad Autónoma de Barcelona,
Bellaterra, 08193, Spain*

**E-mail: diego.pavon@uab.es*

S. DEL CAMPO and R. HERRERA

Instituto de Física, Pontificia Universidad Católica de Valparaíso, Chile

We explore whether the generalized second law of thermodynamics is fulfilled in the transition from a generic initial Einstein static phase to the inflationary phase, with constant Hubble rate, and from the end of the latter to the conventional thermal radiation dominated era of expansion. As it turns out, the said law is satisfied provided the radiation component does not contribute largely to the total energy of the static phase.

Keywords: Early universe; Thermodynamics

1. Introduction

Different cosmological scenarios have been devised to evade the initial singularity of the big bang standard model. These include bouncing universes and the emergent universe. Here we shall focus on a representative of the latter put forward by Ellis and Maartens.¹ In this scenario the initial singularity is replaced by an Einstein static phase in which the scale factor of the Friedmann-Robertson-Walker metric does not vanish and, accordingly, the energy density, pressure and so on stay finite. Thus, the Universe starts expanding from the said phase, smoothly joins a stage of exponential inflation followed by standard reheating to subsequently approach the classical thermal radiation dominated era of the conventional big bang cosmology. Figure 1 depicts this evolution. Fairly generally, the static phase is dominated pressureless matter and radiation (subscripts m and γ , respectively), curvature (which has to be positive, $k = +1$) and a scalar field. Thus the total energy density in this phase (subscript I) obeys, $\rho_{m,I} + \rho_{\gamma,I} + (1/2)\dot{\phi}_I^2 + V_I = \frac{3k}{8\pi G a_I^2}$. In this scenario the potential must be asymptotically flat in the infinite past,

$$V(\phi) \rightarrow V_I \quad \text{as} \quad \phi \rightarrow -\infty, \quad t \rightarrow -\infty, \quad (1)$$

and fall toward a minimum at some finite value. Accordingly, the field rolls down from the static state at $-\infty$ and the potential slowly decreases from its initial value, V_I , in the infinite past. To have acceleration the inequality $V(\phi) - \dot{\phi}^2 > 0$ ought to be fulfilled. Since $V(\phi)$ decreases and $\dot{\phi}^2$ augments, at some time, say $t = t_e$, inflation terminates, then ϕ oscillates about the minimum and reheating takes place, the latter followed by the radiation dominated era.

As demonstrated by Bekenstein, the entropy of a black hole plus the entropy of its surroundings cannot diminish.² This law, aptly named the “generalized second law” (GSL) of thermodynamics as it considers conventional matter/fields and an

event horizon, was extended by several authors to cosmological settings in which the black hole horizon is replaced by a causal cosmic horizon.³ This version of the said law establishes that the entropy of the horizon plus the entropy of the matter and fields within the horizon can never decrease. In this note we study which constraints (if any) the GSL imposes on the two intermediate phases, i.e., from the static phase to exponential inflation and from the reheating to thermal radiation domination (in the static, inflationary, and thermal radiation dominated phases the GSL is trivially fulfilled; see⁴ for details). As causal horizon we shall consider the apparent horizon of area $\mathcal{A} = 4\pi\tilde{r}_A^2$ where $\tilde{r}_A = 1/\sqrt{H^2 + ka^{-2}}$ denotes its radius.⁵ Notice that neither the particle horizon nor the event horizon exist in the static phase; only the particle horizon is meaningful in all the phases considered here. Neglecting quantum effects, the horizon entropy can be written as $S_A = k_B \frac{\mathcal{A}}{4\ell_{pl}^2}$. Assuming the scalar field is in a pure quantum state, the GSL reads $S'_A + S'_m + S'_\gamma \geq 0$, where the prime means derivative with respect the scale factor, a .

2. The GSL at the transitions phases

In the transition from the static to the inflationary phase ($a_I < a < a_{inf}$, see Fig. 1) one has $S'_A = k_B/(2\ell_{pl}^2)H\mathcal{A}^2(\rho + p)/a > 0$, as well as

$$S'_m = -3k_B \frac{N}{a_I^3} \tilde{r}_A^5 \left(H H' - \frac{k}{a^3} \right), \quad T_\gamma S'_\gamma = 2\pi(1 + w_\gamma)(1 + 3w)\tilde{r}_A^3 \frac{\rho_\gamma}{a}, \quad (2)$$

where $w_\gamma = p_\gamma/\rho_\gamma$ and $w = p/\rho$ is the equation of state parameter of the overall fluid -including the scalar field.

With the help of the Einstein field equation

$$H H' - \frac{k}{a^3} = -4\pi G \frac{\rho + p}{a}, \quad (3)$$

it follows that the GSL is fulfilled provided the upper bound

$$\frac{\rho_\gamma}{\rho} \leq \frac{3}{4} \frac{k_B G \pi (1 + w) T_{\gamma I} a_I \left[\frac{4}{\ell_{pl}^2} + 6 \frac{N}{a_I^2} \right]}{|1 + 3w|}, \quad (4)$$

on the amount of radiation energy in the stationary phase is met -see⁴ for details.

The transition between the period of exponential inflation and the thermal radiation dominated phase begins at $a = a_e$ and ends when the products of the inflaton decay at reheating get fully thermalized. In this phase, $H' < 0$, the pressureless matter particles have essentially disappeared, and again $S'_A > 0$. In its turn, the entropy of the mixture of radiation and relativistic particles originated in the decay of the inflaton is given by Eq. (2.2), with w and w_γ replaced by \tilde{w} which is positive-definite. (The tilde is to reminds us that the mixture is not thermalized though $\tilde{w} \rightarrow w_\gamma = 1/3$ as the thermalization process goes on). Thus, in this transition the GSL is guaranteed.

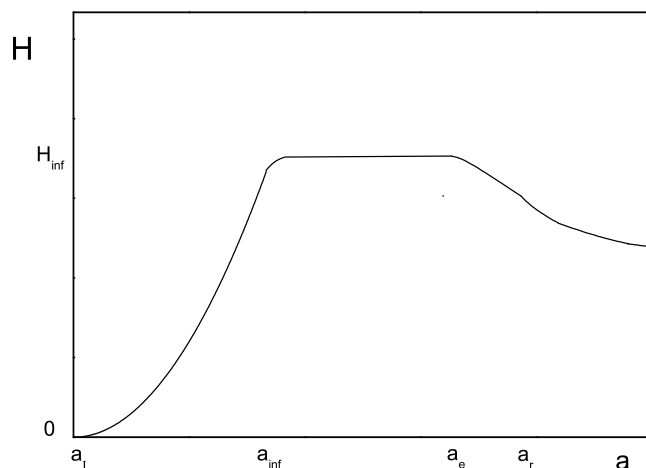


Fig. 1. Schematic evolution of the Hubble function from the Einstein static era to the thermal radiation era. Here a_{inf} and a_e stand for the scale factor at the beginning and end of exponential inflation, respectively; a_r denotes the scale factor at some generic point at the radiation dominated expansion era.

3. Conclusions

For a cosmological model to be worthy of consideration, aside from passing the observational tests, it must comply with thermodynamics; more specifically, it must respect the GSL. We have shown that the toy model of Ref.¹ is thermodynamically safe provided that the radiation energy does not contribute largely to the static phase. Finally, it should be explored if this nice feature is also present in other emergent scenarios.

Acknowledgments

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